Math 206 (Multivariable) - Exam 1 Review

College of St. Catherine — **4 March 2003**

1. Let $\mathbf{A} = (0, 1, 2)$ and $\mathbf{B} = (1, 2, 0)$
   (a) Give a parametrization of the line containing both $\mathbf{A}$ and $\mathbf{B}$, then give a symmetric representation of the line.
   (b) Find the equation of the plane which is orthogonal to this line such that the plane contains the origin.
   (c) Find the distance from the point $\mathbf{C} = (2, 3, 4)$ to the line found in part (a) and to the plane found in part (b).
2. Let $\mathbf{a} = (-1, -3, 5)$. Find a vector $\mathbf{b} = (b_1, b_2, 0)$ that is perpendicular to $\mathbf{a}$ and a vector $\mathbf{c} = (0, c_2, c_3)$ that is perpendicular to $\mathbf{a}$. What is the angle between $\mathbf{b}$ and $\mathbf{c}$?
3. Let $\mathbf{A} = (4, 7, -11)$, $\mathbf{B} = (-1, 2 - 1)$, and $\mathbf{C} = (0, 1, 1)$,
   (a) Find an equation of the plane containing the three points, $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$
   (b) Find the area of the triangle which has its vertices at the points, $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$.
4. Let $\mathbf{D} = (2, -3, 2)$, $\mathbf{E} = (1, -4, 3)$, and $\mathbf{F} = (0, 0, a)$. Find all values of a such that the triangle formed by $\mathbf{D}$, $\mathbf{E}$, and $\mathbf{F}$ is a right triangle. (**Hint:** use the dot product)
5. Consider the set in $\mathbb{R}^3$ such that $x^2 + y^2 \leq 4$. Describe this set using cylindrical and then spherical coordinates.
6. Consider the set in $\mathbb{R}^3$ such that $x^2 + y^2 + z^2 \leq 9$. Describe this set using cylindrical and then spherical coordinates.
7. Consider the function $F(x, y) = \ln(\sqrt{25 - x^2 - y^2})$
   (a) Sketch the following level curves of this function $z = 0$, $z = \ln(3)$, $z = \ln(4)$, and $z = \ln(5)$.
   (b) Find the domain and range of this function.
   (c) What is $\lim_{(x,y)\to(0,0)} F(x, y)$?
8. Sketch and describe the surface
   $$y^2 - 4 = x^2 + z^2$$
   Include a description or sketch of the slices of the surface which are parallel to the three coordinate planes.
9. Give an example of a function $F(x, y)$ which is not defined at the origin, but such that
   $$\lim_{(x,y)\to(0,0)} F(x, y) = 1$$
10. Compute the following limits or show that they do not exist
    (a) $\lim_{(x,y)\to(0,0)} \dfrac{x^3}{x^2 + y^2}$
    (b) $\lim_{(x,y)\to(0,0)} \dfrac{x^2}{x^2 + y^2}$